Math 250 – Notes: Sect. 4.5 – Integration by Substitution

Recall: To take the DERIVATIVE of a **composition of functions**, we used the **<u>chain rule</u>**.

-example- $\frac{d}{dx}(x^2-3)^4 =$

This means:

Practice: Find each derivative. Then, write a corresponding integral to represent the relationship.

1. $y = \sin^3 x$

2. $y = \dfrac{4}{\sqrt{2x-3}}$

3. $y = \tan(x^2)$

Notice that taking the derivative using the chain rule creates a PRODUCT. To integrate a product that was created this way, we need to find a way to "undo" the chain rule . . .

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The process used to evaluate these types of integrals involves a *change of variables*, and is called "integration by substitution."

-example- Evaluate: $\int \sqrt{4x+1}\,dx$

Step 1: Identify the "inner" function of a composition. Call this *u*. **Step 2**: Find *du*. **Step 3**: SUBSTITUTE everything in the original integral with an expression involving *u* or *du*. INTEGRATE this new integral. **Step 4**: Substitute BACK the original *x* expression for *u*.

GENERALIZED Integration Formulas:

1. Power Rule:

2. Trig Functions:

-examples- Evaluate each integral.

a. $\int x\sin(x^2)\,dx$

b. $\int \frac{1}{(3x-5)^2}dx$

c. $\int \frac{3x^2}{\sqrt{x^3+4}}dx$

d. $\int \sin x\sqrt{\cos x}\,dx$

e. $\int \sec^2(1-x)dx$

*f. $\int x\sqrt{x+1}\,dx$

g. Compare/contrast these two integrals: A. $\int x\sqrt{x^2+1}\,dx$ vs. B. $\int \sqrt{x^2+1}\,dx$

h. Which of these integrals can be evaluated using the substitution method?

1. $\int \sin(3x)\,dx$
2. $\int \sin(x^2)\,dx$
3. $\int x\sin(x^2)\,dx$
4. $\int \sin^2 x\,dx$
5. $\int \sin^2 x\cos x\,dx$